MULTIMEDIA UNIVERSITY

STUDENT ID NO

MULTIMEDIA UNIVERSITY

FINAL EXAMINATION

TRIMESTER 2, 2015/2016

TCT 2561 – COMPLEXITY THEORY
(All sections / Groups)

9 MARCH 2016
9:00 a.m. – 11:00 a.m.
(2 Hours)

INSTRUCTIONS TO STUDENTS

1. This Question paper consists of **5 pages** only including the cover page.
2. Attempt **ALL** questions.
3. All questions carry equal marks and the distribution of the marks for each question is given.
4. Please print all your answers **CLEARLY** in the Answer Booklet provided.

Question 1 (2+2+6+5 marks)

(a) Differentiate between computational complexity theory and computability theory.

(b) Differentiate between deterministic Turing machine and non-deterministic Turing machine.

(c) Draw a graph to illustrate each of the following asymptotic equations.

i. $f(n) = O(g(n))$
ii. $f(n) = \Omega(g(n))$
iii. $f(n) = \Theta(g(n))$

(d) Consider the following algorithm for the Tower of Hanoi problem with `n` discs and three poles: `src` is the source, `spare` is the spare, and `dest` is the destination.

```
void towerOfHanoi(int n, char src, char spare, char dest)
{
      towerOfHanoi (n - 1, src, dest, spare);
      towerOfHanoi (1, src, spare, dest);
      towerOfHanoi (n - 1, spare, src, dest);
}
```

i. Write down the recurrence relation of `towerOfHanoi()`.
ii. What is the time complexity class of `towerOfHanoi()` and why?

Continued ....

Question 2 (3+8+4 marks)

(a) Savitch's theorem says that any non-deterministic Turing machine that uses $f(n)$ space can be converted to a deterministic Turing machine that uses only $f^2(n)$ space.
   i. Give the formal definition of Savitch's theorem.
   ii. What is the significance of Savitch's theorem?
   iii. What is the implication on the time complexity?

(b) Given the following definition.

$PATH$ = { $\langle G,s,t \rangle$ | G is a directed graph that has a directed path from s to t }.

   i. Give a high-level Turing machine description for $PATH$ in exponential time.
   ii. Give a high-level Turing machine description for $PATH$ in polynomial time.
   iii. Is $PATH$ problem in time complexity class P? Why?

(c) Given the following definitions.

$TQBF$ = { $\langle \phi \rangle$ | is a true fully quantified Boolean formula }.

$FORMULA\text{-}GAME$ = { $\langle \phi \rangle$ | Player E has a winning strategy in the formula game associated with ϕ }.

Show that *FORMULA-GAME* is PSPACE-complete.

Continued ....

Question 3 (8+3+4 marks)

(a) Give a complexity class example for each of the following computational models and then briefly explain your reason.
   i. Boolean circuit
   ii. Probabilistic Turing machine
   iii. Alternation
   iv. Interactive proof system

(b) Is NPSPACE $\subseteq$ TIME(2^{n^k})? Explain your decision.

(c) Draw a Venn diagram that depicts the relationship between NP-complete, NPSPACE, NP, PSPACE, and NP-hard complexity classes. Label the complexity classes clearly in your drawing.

Continued ....

Question 4 (3+3+5+4 marks)

(a) In your own words, explain the theorem, "if $A \leq_P B$ and $B \in P$ then $A \in P$".

(b) Draw a figure to illustrate mapping reducibility.

(c) Examine the following definitions.

SORTING = { $\langle A[],n \rangle$ | $A[]$ is an array of integers and n is the array size such that we have the array of integers in ascending order }.

DISTINCT = { $\langle A[],n \rangle$ | $A[]$ is an array of integers and n is the array size such that we have distinct integers in the array }.

Construct a polynomial time reduction from *SORTING* to *DISTINCT*.

(d) Describe two methods to prove that a problem B is NP-complete.

End of Page